

Economics 704a Lecture 11: Liquidity Trap and Unconventional Policy II

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Outline: Questions on the Liquidity Trap

1. What Is the Effect of a Liquidity Trap in the NK Model?
2. What Is Optimal Monetary Policy in a Liquidity Trap?
 - 2.1 Forward Guidance (Gali 5.4)
 - 2.2 Other Unconventional Policies
 - 2.3 Is Zero the Lower Bound?
3. What Is the Role of Fiscal Policy in a Liquidity Trap?

What Is the Effect of a Liquidity Trap in the NK Model?

- Start with standard NK model with no cost-push shocks:

$$\pi_t = \beta E_t \{ \pi_{t+1} \} + \kappa x_t$$

$$x_t = E_t \{ x_{t+1} \} - \sigma E_t \{ i_t - \pi_{t+1} - r_{t+1}^n \}$$

- Optimal monetary policy is to set $i_t = r_{t+1}^n$ so $x_t = 0$ and $\pi_t = 0$ (divine coincidence).
- Thought experiment we will use repeatedly today:
 - The natural rate is at its steady state of ρ until period $t - 1$.
 - At period t , learn r_{t+1}^n will follow deterministic path:

$$r_{t+1}^n = \begin{cases} -\Delta < 0 & \text{from } t \text{ to } t + T \\ \rho & \text{from } t + T + 1 \text{ on} \end{cases}$$

- For now, Central Bank pursues optimal discretionary policy
 - Prior to t and from $t + T + 1$ onwards,
set $x_t = -\frac{\kappa}{\vartheta} \pi_t \Rightarrow i_t = \rho \Rightarrow \pi_t = 0$.
 - From t to $t + T$, lower i_t to ZLB so $i_t = 0$.

What Is the Effect of a Liquidity Trap in the NK Model?

- Iterating forward we have:

$$x_t = -\sigma E_t \left\{ \sum_{s=0}^{\infty} \left[\left(\hat{i}_{t+s} - \hat{\pi}_{t+s+1} - \hat{r}_{t+s+1}^n \right) \right] \right\}$$

$$\pi_t = E_t \left\{ \sum_{s=0}^{\infty} \beta^s \kappa x_{t+s} \right\}$$

- Deterministic path so can drop expectations. Split into two sums, one from 0 to T and one from $T+1$ to ∞ :

$$x_t = -\sigma \sum_{s=0}^T \left(\hat{i}_{t+s} - \hat{\pi}_{t+s+1} - \hat{r}_{t+s+1}^n \right) - \underbrace{\sigma \sum_{s=T+1}^{\infty} \left(\hat{i}_{t+s} - \hat{\pi}_{t+s+1} - \hat{r}_{t+s+1}^n \right)}_{\text{Zero By Divine Coincidence}}$$

$$\pi_t = \sum_{s=0}^T \beta^s \kappa x_{t+s} + \underbrace{\sum_{s=T+1}^{\infty} \beta^s \kappa x_{t+s}}_{\text{Zero By Divine Coincidence}}$$

What Is the Effect of a Liquidity Trap in the NK Model?

- Plugging in optimal policy in liquidity trap of $i_t = 0$ and $r_{t+1}^n = -\Delta$, we have:

$$x_t = -\sigma \sum_{s=0}^T (\Delta - \pi_{t+s+1})$$
$$\pi_t = \sum_{s=0}^T \beta^s \kappa x_{t+s}$$

- This implies persistent slump with $x_t < 0$ and $\pi_t < 0$!
 - Start in period $t + T$. Know $\pi_{t+T+1} = 0$ and $\Delta > 0$, so $x_{t+T} < 0$ and $\pi_{t+T} < 0$.
 - In period $t + T - 1$, $\pi_{t+T} < 0$ and $\Delta > 0$, so $x_{t+T-1} < x_{t+T} < 0$ and $\pi_{t+T-1} < \pi_{t+T} < 0$.
 - Working backward, $x < 0$ and $\pi < 0$ all the way back to period t , with bigger output gaps and deflation farther back.

What Is the Effect of a Liquidity Trap in the NK Model?

- Why the big slump?
- Even if inflation were zero, consumption would be depressed by

$$x_t = -\sigma \sum_{s=0}^T \Delta$$

- Households are saving “too much” because r_t is “too high.”
- Key Idea: Deflation exacerbates the ZLB.
 - Deflation occurs because negative output gaps push down MC.
 - This pushes r_t higher as $r_t = -E_t \{\pi_{t+1}\}$, which makes x_t lower, leading to more deflation....

$$x_t = -\sigma \sum_{s=0}^T (\Delta - \pi_{t+s+1})$$

- Inflation is forward looking, so deflation is worst at the beginning and then gets better due to forward-looking sum.

The Paradox of Flexibility

- Would more flexible prices make things better?
 - NO! Surprisingly, they make things *worse*!
- Output gap with perfectly sticky prices is:

$$x_t = -\sigma \sum_{s=0}^T \Delta$$

- Output gap with flexible prices (larger κ) is:

$$x_t = -\sigma \sum_{s=0}^T (\Delta - \pi_{t+s+1})$$

with π_{t+s+1} becoming more negative as $\kappa \rightarrow \infty$.

- Intuition: Deflation is what turbocharges liquidity trap.
 - More flexibility \Rightarrow more deflation \Rightarrow worse spiral?

The Paradox of Flexibility

- Why does model not return to flex price model when κ hits ∞ ?
- Because even in that flexible price case, we do not get to the flexible price equilibrium
 - Only the flexible price equilibrium if *all* prices, including real interest rate, can get to their flexible price levels
 - But in the Paradox of Flexibility case, the real interest rate is stuck away from the natural interest rate
- In other words, flexibility is worse *conditional on* $r_t \neq r_t^n$

What Is Optimal Monetary Policy in a Liquidity Trap?

- What does monetary policy want to do in a liquidity trap?
 - $i_t = 0 \Rightarrow$ It can't do anything!
- But, as with optimal monetary policy, can gain from committing self to a non-discretionary solution.
 - This time with respect to policy *after* the liquidity trap.
 - In particular, it wants to *commit to inflating*!

What Is Optimal Monetary Policy in a Liquidity Trap?

- In T period liquidity trap with commitment:

$$x_t = -\sigma \sum_{s=0}^T (\Delta - \pi_{t+s+1}) - \sigma \sum_{s=T+1}^{\infty} (i_{t+s} - \pi_{t+s+1} - \rho)$$

$$\pi_t = \sum_{s=0}^{\infty} \beta^s \kappa x_{t+s}$$

- Causing an inflationary boom when the liquidity trap is over:
 1. Reduces “over saving” problem causing the trap.
 - Boom in future, so less reason to save.
 - This is the root cause. It would help even with fixed prices.
 2. Reduces deflation \Rightarrow mitigates deflationary spiral.
 - Inflation today pushes r_t down towards r_t^n .

Central Bank Problem

$$\min_{\{x_{t+s}, \pi_{t+s}\}} \frac{1}{2} \sum_{s=0}^{\infty} \beta^s (\pi_{t+s}^2 + \vartheta x_{t+s}^2) \quad \text{s.t.}$$

$$\pi_t = \beta \pi_{t+1} + \kappa x_t$$

$$x_t \leq x_{t+1} + \sigma (\pi_{t+1} + r_{t+1}^n)$$

- Second constraint combines IS and ZLB.
 - Recall when ZLB does not bind, we choose $\{x, \pi\}$ subject to NKPC and use DIS to back out i that implements this allocation. So if $i = 0$, second constraint is an inequality.
 - If $i = 0$, however, we need NKPC to determine agg demand in liquidity trap and $\{x, \pi\}$, so it binds with equality.
- T -period liquidity trap as before:

$$r_{t+1}^n = \begin{cases} -\Delta & \text{from } t \text{ to } t+T \\ \rho & \text{from } t+T+1 \text{ on} \end{cases}$$

Central Bank Lagrangian

$$\mathcal{L} = \frac{1}{2} \sum_{s=0}^{\infty} \beta^s \left[\begin{aligned} &(\pi_{t+s}^2 + \vartheta x_{t+s}^2) + \xi_{1,t+s} (\pi_{t+s} - \kappa x_{t+s} - \beta \pi_{t+s+1}) \\ &+ \xi_{2,t+s} (x_{t+s} - x_{t+s+1} - \sigma (\pi_{t+s+1} + r_{t+s+1}^n)) \end{aligned} \right]$$

- FOCs:

$$\pi_t + \xi_{1,t} - \xi_{1,t-1} - \frac{\sigma}{\beta} \xi_{2,t-1} = 0$$

$$\vartheta x_t - \kappa \xi_{1,t} + \xi_{2,t} - \frac{1}{\beta} \xi_{2,t-1} = 0$$

- Complementary slackness conditions:

$$\xi_{2,t} \geq 0, \quad i_t \geq 0, \quad \xi_{2,t} i_t = 0$$

- Interpretation: $i_t = 0$ and dynamic IS binds or $i > 0$ and off dynamic IS for allocation, as on previous slide.
- Multiplier is always positive if constraint binds.

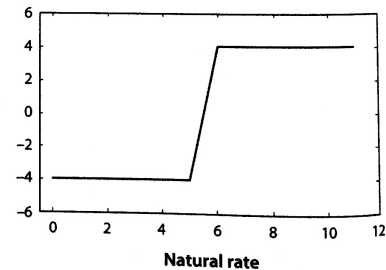
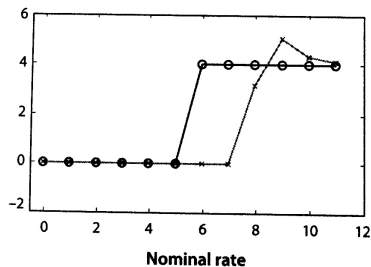
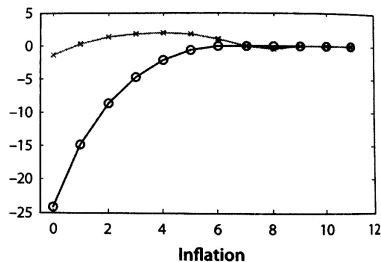
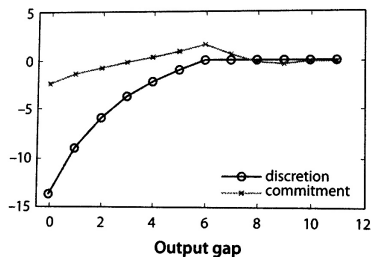
Central Bank Commitment Solution

- Show positive output gap and inflation at $T + 1$ by differencing FOCs for between $T + 1$ and T and using $\xi_{2,T+1} = 0$:

$$x_{T+1} - x_T = -\frac{\kappa}{\vartheta}\pi_{T+1} + \frac{\beta + \sigma\kappa}{\beta\vartheta}\xi_{2,T} + \frac{1}{\beta\vartheta}(\xi_{2,T} - \xi_{2,T-1})$$

- First term is standard leaning against the wind effect. If this alone, $\pi_{T+1} = x_{T+1} = 0$.
- Offset by second two terms with $\xi_{2,T} > 0$ which make you want to set $x_{T+1} > 0$.
- Asymptotically returns to $x_t = \pi_t = 0$.
- Intuition: Second order inflation and output gap loss in future, first order output gap and deflation gain today.
- Werning (2012) solves full dynamic path using continuous time methods, shows $i_t = 0$ for $t \in [t_0, T_c]$ for $T_c > t + T$ and then jumps discretely.

Central Bank Commitment Solution



Central Bank Commitment Solution in Practice

- This commitment solution motivates *forward guidance*.
 - Announce you are going to keep your rate low for a long time, unconditional on market conditions.
 - Idea: Get people to believe that you will keep rates low after the ZLB does not bind.
 - Problem: Not a time consistent commitment.
 - Also when do you know you are out? T -period trap is stylized and period of ending is endogenous.
- Fed used forward guidance:
 - In December 2008 says “likely to warrant exceptionally low levels of the federal funds rate for some time.”
 - In August 2011, introduce specific date stating that will be low through mid 2013.
 - Pushed that out twice to late 2014 and mid-2015 in 2012.

Other Unconventional Policies

- Fed also pursued *Large-Scale Asset Purchases* (also known as “quantitative easing”), which lie outside the basic NK model.
- There is not just one short term rate.
 - Fed Funds Rate
 - Longer-maturity treasury rates.
 - Checking interest / certificate of deposit rate.
 - Mortgage rates.
 - Business loan rates.
- Other rates are usually spreads over FFR.
- By buying treasuries and GSE mortgage-backed securities, reduce spreads and interest rates for consumers and businesses.
 - And perhaps stimulate the housing market directly by expanding mortgage credit?

Optimal Inflation Rate?

- Many of these policies are controversial.
- Most controversial: raise the inflation target above 2 %.
 - Benefits:
 1. Helps us get out of liquidity trap now.
 2. In future, need $r_t^n < -\pi_t$ to fall in, so fall in less frequently.
 - Olivier Blanchard floated 4%. Nobody has yet adopted.
 - Worry about runaway inflation and that inflation expectations will become “unanchored.”

Is Zero the Lower Bound?

- Recently, moved into a world of negative interest rates.
 - Swiss, Swedes, Danish, ECB, and Japanese all had negative rates (for banks, not people).
 - Get banks to lend money by taxing reserves and reducing other rates to zero.
- Money demand has not exploded up yet.
 - Clearly would if you go negative enough.
 - But what is “negative enough”? We really do not know...
- See Rognlie (2016) for model where money demand explodes at negative rate rather than zero, but negative rates cause costly distortions.
- See Eggertsson et al. (2019) for evidence that the pass-through of policy rates to deposit rates breaks down when the policy rate becomes negative.

What Is the Role of Fiscal Policy in a Liquidity Trap?

- In 2009, passed ARRA (the “Stimulus Act”).
- In there a stronger case for fiscal stimulus at the ZLB?
 - Is the multiplier higher?
 - Other justification: $x_t < 0 \Rightarrow$ marginal costs are low, so cheap for government to buy its goods now, and i_t is low so cheap for it to finance with bonds.
- To answer multiplier question, first look at multiplier in normal times in standard NK model, then consider ZLB.

Government Spending in New Keynesian Model

- Assume government consumes G_t and finances with lump sum taxes T_t and bonds B_t :

$$\frac{1}{R_{t+1}} B_{t+1} = B_t + G_t - T_t$$

- With perfect capital markets and no default

$$B_t + \sum_{s=0}^{\infty} \frac{G_{t+s}}{\prod_{j=0}^s R_{t+1+j}} = \sum_{s=0}^{\infty} \frac{T_{t+s}}{\prod_{j=0}^s R_{t+1+j}}$$

- Assume G_t follows exogenous process.

Ricardian Equivalence

- Household BC:

$$\begin{aligned}C_t &= \frac{W_t}{P_t}N_t + TR_t + PR_t - T_t - \frac{B_t - Q_{t-1}B_{t-1}}{P_t} - \frac{M_t - M_{t-1}}{P_t} \\&= Income_t - T_t - \frac{B_t - Q_{t-1}B_{t-1}}{P_t} - \frac{M_t - M_{t-1}}{P_t}\end{aligned}$$

where $Income_t = \frac{W_t}{P_t}N_t + TR_t + PR_t$.

- Then present value BC is:

$$B_t + M_t + \sum_{s=0}^{\infty} \frac{Income_{t+s} - T_{t+s}}{\prod_{j=0}^s R_{t+1+j}} = \sum_{s=0}^{\infty} \frac{C_{t+s}}{\prod_{j=0}^s R_{t+1+j}}$$

and substituting government BC, Ricardian equivalence holds.

- Timing of taxes does not matter.
- But changes in G_t matter.

Equilibrium

$$\frac{W_t}{P_t} = \frac{\chi N_t^\varphi}{C_t^{-\gamma}}$$

$$1 = \beta E_t \left\{ Q_t \frac{P_t}{P_{t+1}} \frac{C_{t+1}^{-\gamma}}{C_t^{-\gamma}} \right\}$$

$$P_t = [\theta P_{t-1}^{1-\varepsilon} + (1-\theta) P_t^{*1-\varepsilon}]^{\frac{1}{1-\varepsilon}}$$

$$P_t^* = (1+\mu) E_t \left\{ \sum_{s=0}^{\infty} \frac{\theta^s \Lambda_{t,t+s}^n P_{t+s}^\varepsilon Y_{t+s}}{\sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k}^n P_{t+k}^\varepsilon Y_{t+k}} \frac{W_{t+s}}{A_{t+s}} \right\}$$

$$Y_t = C_t + G_t$$

$$Y_t = A_t N_t \left[\int_0^1 \left(\frac{N_t(i)}{N_t} \right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

Log Linearized IS and Natural Rate

- Log linearizing resource constraint gives:

$$\hat{y}_t = (1 - s_g) \hat{c}_t + s_g \hat{g}_t$$

- Log linearizing Euler and plugging in:

$$\hat{y}_t = - (1 - s_g) \sigma \left(\hat{i}_t - E_t \{ \hat{\pi}_{t+1} \} \right) + E_t \{ \hat{y}_{t+1} \} + s_g (\hat{g}_t - E_t \{ \hat{g}_{t+1} \})$$

- Consequently, in the flex price equilibrium

$$\hat{y}_t^n = - (1 - s_g) \sigma \hat{r}_{t+1}^n + E_t \{ \hat{y}_{t+1}^n \} + s_g (\hat{g}_t - E_t \{ \hat{g}_{t+1} \})$$

- Differencing gives modified IS curve:

$$\tilde{y}_t = - (1 - s_g) \sigma E_t \left\{ \hat{i}_t - \hat{\pi}_{t+1} - r_{t+1}^n \right\} + E_t \{ \tilde{y}_{t+1} \}$$

where:

$$\hat{r}_{t+1}^n = \frac{1}{(1 - s_g) \sigma} (E_t \{ y_{t+1}^n \} - \hat{y}_t^n) + \frac{s_g}{(1 - s_g) \sigma} (\hat{g}_t - E_t \{ \hat{g}_{t+1} \})$$

Flex Price Equilibrium

$$\begin{aligned}\hat{y}_t^n &= \hat{a}_t + \hat{n}_t^n \\ \hat{y}_t^n - \hat{n}_t^n &= \varphi \hat{n}_t^n + \gamma \hat{c}_t^n \\ \hat{y}_t^n &= (1 - s_g) \hat{c}_t^n + s_g \hat{g}_t\end{aligned}$$

- Combining with r_t^n :

$$\hat{y}_t^n = \left(\frac{1 + \varphi}{\varphi + \frac{\gamma}{1 - s_g}} \right) \hat{a}_t + \gamma \frac{s_g}{(1 - s_g) \varphi + \gamma} \hat{g}_t$$

- Consequently,

$$\hat{r}_{t+1}^n = -\psi_a (\hat{a}_t - E_t \{\hat{a}_{t+1}\}) + \psi_g (\hat{g}_t - E_t \{\hat{g}_{t+1}\})$$

$$\text{where } \psi_a = \frac{\gamma}{(1 - s_g) \varphi + \frac{\gamma}{1 - s_g}} \text{ and } \psi_g = \frac{\gamma s_g}{(1 - s_g) \varphi + \frac{\gamma}{1 - s_g}}.$$

Summary of Model With G

$$\tilde{y}_t = -(1 - s_g) \sigma E_t \left\{ \hat{i}_t - \hat{\pi}_{t+1} - r_{t+1}^n \right\} + E_t \{ \tilde{y}_{t+1} \}$$

$$\hat{\pi}_t = \kappa \tilde{y}_t + \beta E_t \{ \hat{\pi}_{t+1} \}$$

$$\hat{y}_t^n = \left(\frac{1 + \varphi}{\varphi + \frac{\gamma}{1 - s_g}} \right) \hat{a}_t + \gamma \frac{s_g}{(1 - s_g) \varphi + \gamma} \hat{g}_t$$

$$\hat{r}_{t+1}^n = -\psi_a (\hat{a}_t - E_t \{ \hat{a}_{t+1} \}) + \psi_g (\hat{g}_t - E_t \{ \hat{g}_{t+1} \})$$

$$\hat{y}_t = \tilde{y}_t + \hat{y}_t^n$$

- Government spending affects \hat{r}_t^n and \hat{y}_t^n
 - $\uparrow \hat{g}_t \rightarrow \uparrow \hat{y}_t^n$ due to neg wealth effect from taxation increasing aggregate supply.
 - $\uparrow \hat{g}_t \rightarrow \uparrow \hat{r}_t^n \rightarrow \downarrow \tilde{y}_t$ due to aggregate demand effects of government spending.

Government Spending Multiplier

- If Central Bank sets $i_t = r_{t+1}^n + E_t \{\pi_{t+1}\}$, $\tilde{y}_t = 0$ and $\hat{y}_t = \hat{y}_t^n$:

$$\frac{dy_t}{dg_t} = \frac{dy_t^n}{dg_t} = \frac{\gamma s_g}{(1 - s_g)\varphi + \gamma}$$

- The multiplier is then:

$$\frac{dY_t}{dG_t} = \frac{Y}{G} \frac{dy_t}{dg_t} = \frac{\gamma}{(1 - s_g)\varphi + \gamma} < 1$$

- CB completely offsets the agg demand effect of gov't spending.
 - Leaves only aggregate supply (wealth) effect.
- If the CB does nothing, additional effect through \tilde{y}_t rising due to effect of gov't purchases on agg demand.
 - Modestly above one in calibrations (no closed form).

Government Spending Multiplier in a Liquidity Trap

- Return to T -period liquidity trap example with CB setting optimal discretionary policy:

$$\begin{aligned}\tilde{y}_t &= \sigma (1 - s_g) E_t \left\{ \sum_{s=0}^T (\pi_{t+s+1} + \hat{r}_{t+s+1}^n) \right\} \\ &= -\sigma (1 - s_g) E_t \left\{ \sum_{s=0}^T [\Delta - \pi_{t+s+1} - \psi_g (\hat{g}_{t+s} - E_t \{ \hat{g}_{t+s+1} \})] \right\}\end{aligned}$$

- Government spending has same stimulative effect as inflation.*
 - Problem is “too much saving” and too little spending.
 - Government spending makes up shortfall, improving output gap through agg demand effect even in absence of inflation effect.
 - Also pushes up wages and marginal costs, creating inflation and mitigating a deflationary spiral.
- Large multipliers at ZLB in calibrated models (well above 1).

Government Spending Multiplier in a Liquidity Trap

- Normal times \Rightarrow use monetary policy (more nimble).
- ZLB \Rightarrow use fiscal policy (monetary policy has hands tied and has to make non-credible commitments).
- However, very large multipliers *depend on inflation responding to fiscal stimulus*.
 - For ZLB episode from Great Recession until pandemic, inflation was fairly anchored.
 - Suggests multiplier may not have risen quite so much.
 - Estimating multiplier in and outside ZLB is source of continued debate.

New Perspectives on the Monetary Transmission Mechanism

- Last two lectures will cover new papers to give you an idea of what frontier research in macro is about.
 - Will feel more like a second year class.
 - Read the papers and be ready to discuss!
- Focus will be on recent papers reevaluating the monetary transmission mechanism.
 - In the New Keynesian model, all about intertemporal substitution by representative consumer.
 - But is this really how monetary policy works?
- Papers will focus on one or more dimensions of heterogeneity.
 - Will build on what you learned with David.

New Perspectives on the Monetary Transmission Mechanism

1. Heterogeneous Agents and Monetary Transmission

- Key paper: Kaplan, Moll, and Violante (2018)
- Secondary Papers: Gali, Lopez-Salido, and Valles (2007), Kaplan and Violante (2014)

2. Household Finance, Housing, and Monetary Policy

- Key paper: Wong (2021)
- Secondary Papers: Di Maggio et al. (2017), Beraja et al. (2019)
- Please read key papers!
 - Last “problem set” is brief response assignment on both of the key papers.